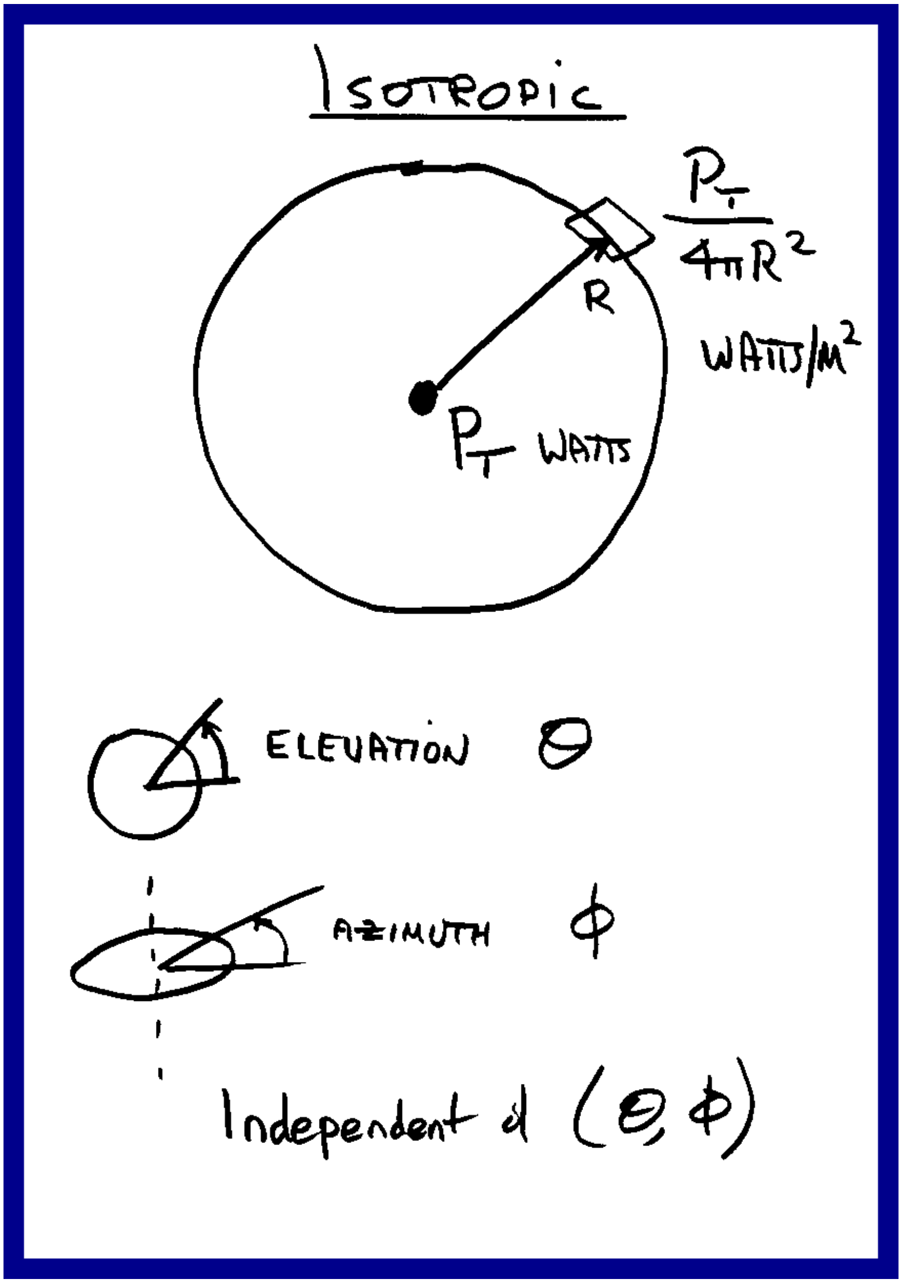
Some Useful Math Basics
And Maths Culture
Contributed by David Jefferies, Ph.D

INTRODUCTION
Jack, I have to write a bit of maths for our short course. Here attached is what I've done so far in 15 gif files. They derive the wave equation, velocity and impedance of free space from Maxwell. Plus, they then go on to discuss the potentials and the Lorentz gauge. They also show how the fields go as 1/r and give a simple Green's function.

Regards
David

PS: ***They're now yours, an Easter Egg...!!***

David's 15 useful math notes follow below with a brief description of their use underneath each.



0001-isotropic

An "isotropic" antenna radiates equally in all directions around the sphere, in both azimuth and elevation. An omnidirectional antenna radiates equally around a plane, usually taken to be the azimuth plane, but its radiation out of this plane varies.

P_D = Power Density (Watt/m^2)

E = Electric Field V/m

H = Magnetic Field A/m.

So $EH = \frac{V\,A}{m^2}$ = Watt/m^2

$\frac{E}{H} = Z_0 = \frac{V/m}{A/m} \equiv$ OHMS

$\boxed{\text{IMPEDANCE OF FREE SPACE } Z_0}$

$Z_0 = 120\pi$ OHMS

= 377 OHMS

0002-impedance

Impedance has units voltage/current, and in free space also has units of (electric field strength)/(magnetic field strength). In a radiating wave in free space, the electric field determines the magnetic field, and vice versa.

E, H are r.m.s.

root mean square

$$EH = P_D$$

$$E/H = Z_0$$

$$E^2 = P_D Z_0$$

$$H^2 = P_D / Z_0$$

<u>ISOTROPIC</u> $P_D = \dfrac{P_T}{4\pi R^2}$

$$E = \sqrt{\frac{P_T Z_0}{4\pi}} \left(\frac{1}{R}\right) \quad \text{V/m}$$

(r.m.s remember)

0003-e-h

In a radiating wave, the electric field falls off proportional to distance from the source, and varies as the square root of the transmitter power

$$H = \frac{E}{Z_0} = \sqrt{\frac{P_T}{4\pi Z_0}}\left(\frac{1}{R}\right)$$

(r.m.s) $\underline{\underline{A/m.}}$

<u>Example</u>

P_T = 1,000 watts

R = 10,000 m = 10 km

Z_0 = 377 Ω (remember?)

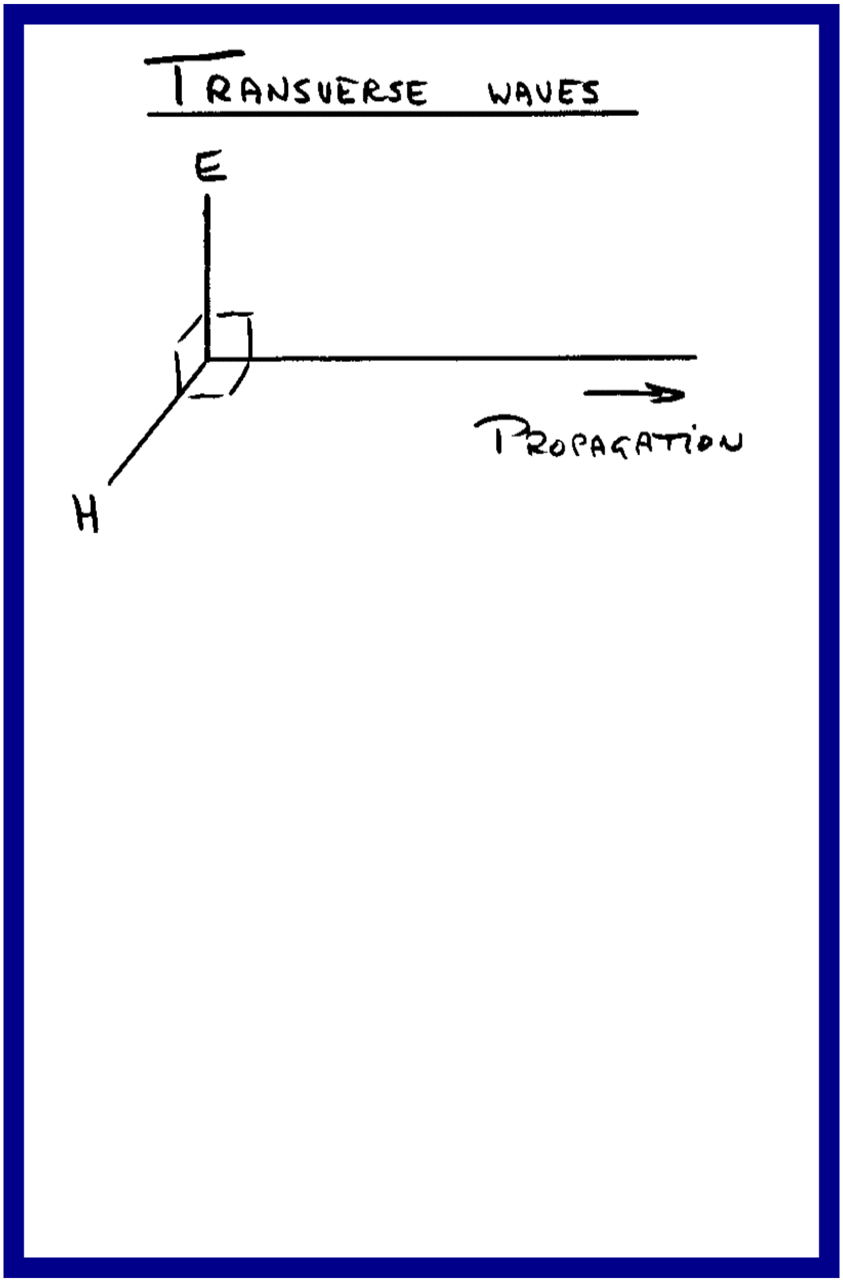
<u>So</u> E = 17.3 mV/m

H = 45.9 μA/m

P_D = Power density = 0.796 μW/m^2

0004-eh-from-power

In a radiating wave, the magnetic field also falls off proportional to distance from the source.



0005-transverse-waves

Radiating electromagnetic waves are "transverse"; the electric field, magnetic field, and propagation direction are all mutually at right angles.

Maxwell's Equations

$$\mathrm{div}\, D = \rho$$

$$\mathrm{curl}\, E = -\frac{\partial B}{\partial t}$$

$$\mathrm{div}\, B = 0$$

$$\mathrm{curl}\, H = \frac{\partial D}{\partial t} + J$$

$$\left.\begin{array}{ll} \rho & \text{Coulombs}/m^3 \\ J & \text{Amps}/m^2 \end{array}\right\} \text{sources}$$

$$\left.\begin{array}{l} D = \epsilon E \\ B = \mu H \end{array}\right\} \text{constitutive relations}$$

0006-maxwells-eqns
Maxwell's equations

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

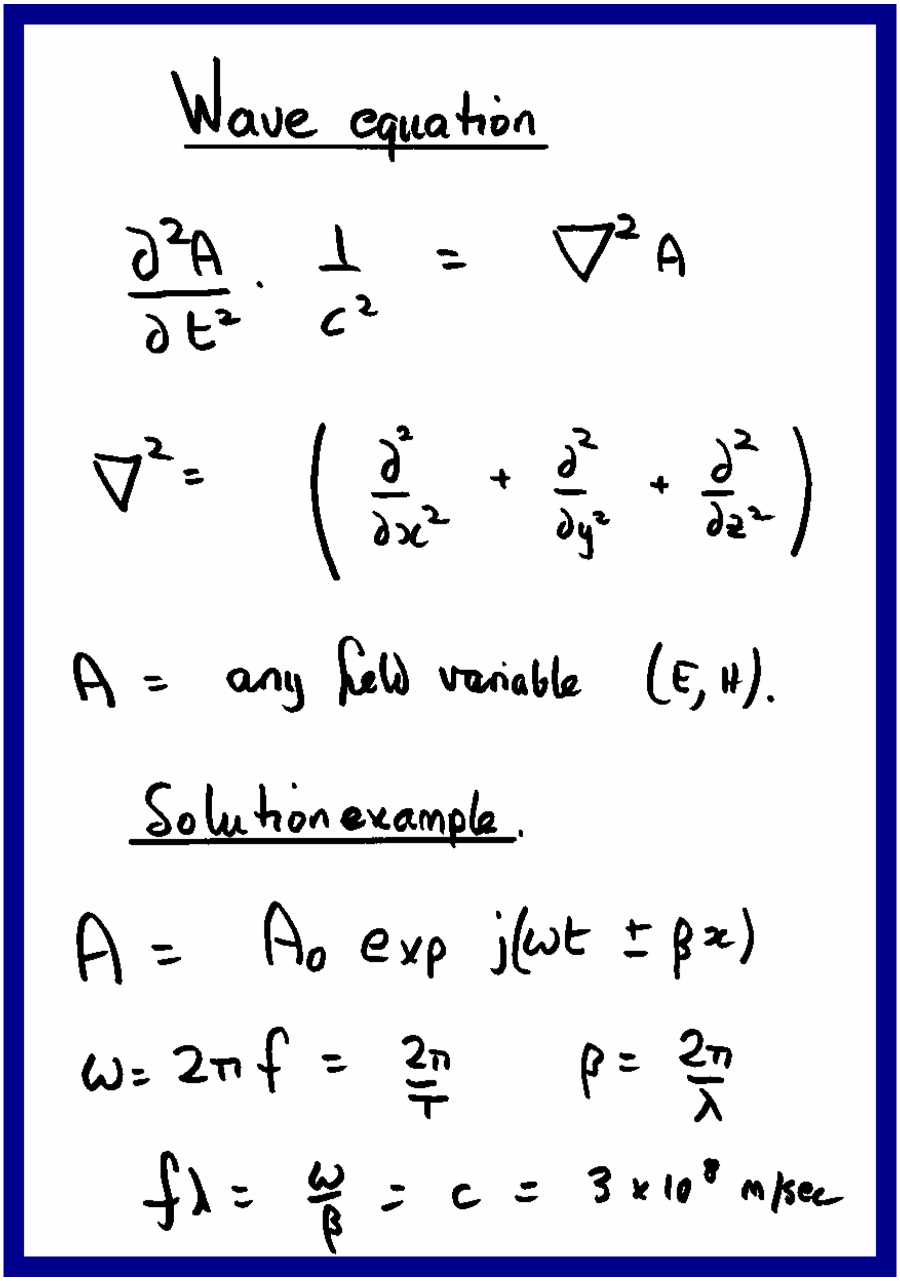
$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \times 10^{8} \text{ m/sec.}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ ohms} = Z_0$$

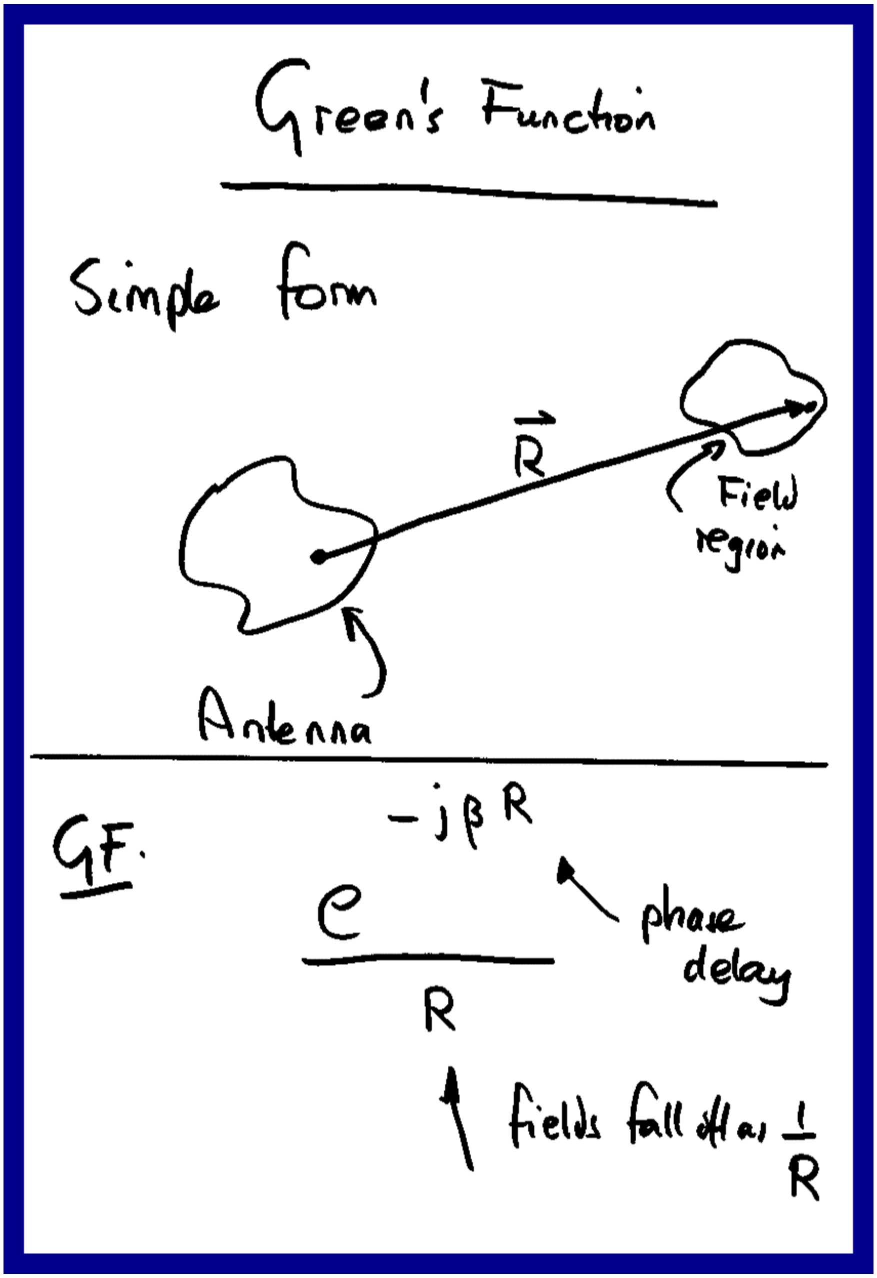
0007-eps0-mu0

Permittivity and permeability of free space



0008-wave-eqn

Wave equation (a 4-dimensional partial differential equation)



0009-greens-function

Radiation Green's function, incorporating the phase delay and the fall off of fields proportional to distance from the source.

Derivation of Wave Equation

In vacuum, no charge or current.

$\rho \quad J = 0$

$$\mathrm{div}(\epsilon_0 E) = 0$$

$$\mathrm{div}\ B = 0$$

$$\mathrm{curl}\ E = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\mathrm{curl}\ H = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t}$$

$$\boxed{\mathrm{curl}\ \mathrm{curl} \equiv \mathrm{grad}\ \mathrm{div} - \nabla^2}$$

$$\mathrm{curl}\ \mathrm{curl}(E) = -\mu_0 \frac{\partial}{\partial t} \mathrm{curl}\ H$$

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

0010-derivation-wave-eqn

Derivation of the wave equation from Maxwell's equations in a vacuum

Space and time derivatives

div, curl, grad are all first space derivatives

If waves propagate as

$$\exp j(\omega t \pm \vec{\beta} \cdot \vec{r})$$

then $\frac{\partial}{\partial t} \equiv$ multiplication by $j\omega$

$$\frac{\partial}{\partial \vec{r}} \equiv j\vec{\beta}\,\cdot$$

$$\text{div} = \left[\frac{\partial}{\partial x}\hat{i},\ \frac{\partial}{\partial y}\hat{j},\ \frac{\partial}{\partial z}\hat{k}\right]$$

$\hat{i}\ \hat{j}\ \hat{k}$ unit vectors along x, y, z

0011-derivatives
Technical definitions of first derivatives

$$\mathrm{Curl}\ E = -\frac{\partial B}{\partial t}$$

$$\nabla \wedge E = -\frac{\partial B}{\partial t}$$

$$j\vec{\beta} \wedge \vec{E} = -j\omega\mu_0 \vec{H}$$

$$\left|\frac{E}{H}\right| = \frac{\omega\mu_0}{\beta}$$

$$\frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c = 3\times10^{8}\ \mathrm{m/s}.$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

$$\frac{\omega\mu_0}{\beta} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \mathrm{ohm}$$

0012-Zo-from-maxwell

Derivation of the impedance of free space from Maxwell's equations

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$= \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\beta^2 E = \mu_0 \epsilon_0 \omega^2 E$$

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \times 10^8 \text{ m/s.}$$

$$\frac{2\pi f}{2\pi/\lambda} = f\lambda = c.$$

0013-velocity-from-maxwell

Derivation of the velocity of electromagnetic radiation, from Maxwell's equations

Vector potential A

$$B = \text{curl } A$$

Scalar potential ϕ

for static fields:-

$$E = -\text{grad } \phi$$

$$= \left[-\frac{\partial \phi}{\partial x}\hat{i} - \frac{\partial \phi}{\partial y}\hat{j} - \frac{\partial \phi}{\partial z}\hat{k} \right]$$

Since $\boxed{\text{div curl} \equiv 0}$

$$\text{div } B = \text{div curl } A \equiv 0$$

automatically satisfied.

0014-potentials
Definitions of vector and scalar potentials

$$\text{curl } H = \frac{\partial D}{\partial t} + J$$

current densit
A/m^2

$$\text{curl } \frac{B}{\mu_0} = \epsilon_0 \frac{\partial E}{\partial t} + J$$

$$\text{curl curl } A = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} + \mu_0 J$$

$$\text{grad div } A - \nabla^2 A = \frac{1}{c^2} \frac{\partial E}{\partial t} + \mu_0 J$$

use $E = -\frac{\partial A}{\partial t} - \text{grav } \phi$

Lorentz Gauge $\text{div } A + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$

0015-lorentz-gauge

Choice of the radiation gauge, Lorentz gauge. div(A) may be chosen as it is only curl(A) which is defined in terms of the magnetic field B

Maths Culture: An Open Discussion Around the Globe

The following is only a small portion an open discussion on the antenna-discussion list about the "Maths Culture" that exists around the world and their subtle to wide differences.

This is one large thread that was not only educational but also entertaining. Most of all, it placed a spotlight on that fact that various countries have different approaches to math convention and its variations of density.

The simpler the better, and what could be more simply expressed but yet mean so much as Einstein's: $E=MC^2$. Maxwell was no slouch at this either.

If you want to enjoy discussions like this and others, you are missing a wonderful resource at the antenna-discussion list sponsored by *antenneX*. More than 2000 of some of the best experts antenna experts in the world read and/or contribute to the discussions, and hardly does a posting for help go unanswered within a few hours. **— *Jack L. Stone, Publisher***

Now, the partial discussion about Maths Culture starting with my own initial post that launched this interesting thread:

I wonder if I may start a thread on "Math Culture" considering that this is an International List which includes all of the "math cultures" I will refer to. Others math cultures are welcomed to be added to the list to review and discuss of course. No offense by leaving any cultures out on this initial posting.

From time to time, our friends from different countries have been concerned about explaining their views correctly in English, not being their native language. I have had the general impression that if we all spoke the language of "Math" about scientific matters, then it broke down any language barriers to the extent we could then all understand a concept equally. Of course the simpler the math expression, the better understood – and that applies to most things.

I was having a personal exchange with a friend in Europe who has a strong background in mathematics and in one such exchange, he made this observation that gave me cause to pause and contemplate. Mind you, these are his views and I present this discussion to learn more about this observation, if it is a real issue as so described by my friend.

I was told the Russians have always been very good at "dense" mathematics, whereas the British and Americans are good at practical engineering and more intuitive ways of looking at things.

I was told that the Italian culture is deeply mathematically inclined, as also is the Greek culture. Further that, often these "math cultures" use the art of dressing up simple ideas to look quite complex. That way they are "overly-presented" in American journals, which tend to like papers couched in math not readily understood according to my friend.

My only purpose for bringing this up is that, if this is true, then one might filter or skew an opinion on this discussion list without these cultural differences in mind -- very much as the selection of the proper or improper word or phrase. My friend thought those using dense math are capable of the most glaring mis-conceptions when it comes to the application to physics.

Thus, math is a tool and it is also a language. From that, we can learn that it is best used when most simply expressed.... no more than necessary and no less -- but, perhaps we still are not speaking in the same exact terms -- this was the surprise to me!

Comments....???

Best regards,
Jack L. Stone

Jack,
In principle, YES! The language of "Math" is similar to the language of "Music"; one allows minds to communicate, the other souls... BUT: If you are looking for a "broad band" Discussion List, then it is wise to find the ideal mixture of English & Maths. Let us name it the language of "Mathenglish". This will allow us to continue keeping together all education levels and faculties of the group. The Math language alone, will create an elite group separating from the rest especially when Maths becomes "too abstract". For clearing scientific questions, as you say, it is almost compulsory to use the language of "Math" what I find salutary. See for example the very eloquent answers to such questions given by Kirk McDonald.

Best regards,
Werner

Thanks, Werner.
Math is "similar" to music but music is not exact. Music is subject to an "emotional" factor whereby some notes must have a varible of emphasis (soft to loud). The same sheet music can be played twice in such a manner that the "song" will not sound like the same one. Imagine how Mozart and the like can be interpreted. Math is not so emotional and each symbol (note) should mean the same thing — with exception that our different math cultures may use a different symbol to mean the same -- but, that is not a problem as long as we all know thise differences.

One more reference to "music." I remember the really funny scene from a Mozart movie, where after a performance by Mozart, the Emperor came to congratulate Mozart. During that scene, Mozart asked the Emperor what he thought of the performance, to which he replied (as it was the only thing he could thing of) "..er, too many notes!). To which the flabbergasted Mozart replied "But it contained just as many notes as was needed: not too many and not too few" That is comparable to math — keep the equations as short and simple as possible.

Further, as a publisher, too much math can lose the reader's attention to an article. Essentially, the author should describe a concept (theory), demonstrate it (tests) and then prove it with math and that it fits with Mother Nature's intentions (laws of the universe).

Any corrections or alternatives to what I have said are welcomed....

Best regards,
Jack L. Stone

Jack,

Music is more like language than math. It has a grammar and a vocabulary, etc. And, just as we understand language when spoken in a wide range of accents, or dialects, we also understand music when performed in a wide variety of styles.

For those who are not deeply immersed in the structure of music, there is a strong mathematical basis underlying it, not only in rhythm, but also in form and structure. I speak, of course, of formally composed music by gifted and trained composers. Not my efforts.

But, the mathematical underpinnings do not imply a precision of meaning. For music, like language, means precisely what the creator of it intends, no more and no less.

It should also be pointed out that some composers, in rebellion against the formal structures of music, have offered us aleatoric, or random, music in which the performer - not the composer - is the main "chooser" of the content.

And as we all know, each generation rebels to some extent against the music of their parents and grandparents. How many
times was I tempted to tell my teenagers to "turn off that noise!"?

I speak to this subject as a former professor of music and head of an electronic music studio... during the early days of such things.

Although we may not know a lot about art, we know what we like. And that, in the end, is all that matters.

Perhaps, in some way, antennas are like that, too.

I had colleagues who could bore us to death with facts about music, but for some of us it's more important that we enjoy it, than understand it. For those who create, deep knowledge is essential. For others who
enjoy their efforts, enjoyment is enough.

To each his own? Room for all?
Kindest regards to all.

Jim Rothwell

Hi, Jim: Yes, when we *hear* the music performed do we understand it. But, not purely from the sheet unless there is a translator (conductor) and that music is scored in the purest, perfect manner. Math is discernable from the sheet alone.

But, I am not qualified to debate with a Prof of Music although I have played numerous instruments and written 50 or so original works and performed on stage (in the 70s). However, I'm self-taught and not good at the Technicals like you have to be.

....sorry for drifting OT.

Best regards,
Jack L. Stone

Jack,

Thanks for your comments from a person who is so accomplished as an "amateur". Pun intended.

I think you make an important point. Some people without formal credentials make as much contribution as others who are thoroughly schooled in a subject. Often the person who doesn't know that something is impossible is able to do it anyway, or bring a fresh viewpoint to it.

Of course, in other instances formal schooling is absolutely needed to solve a problem.

As a supposed "expert" in my field, I always kept on my wall a collection of sayings by "experts" who had by then been proven wrong:

"X-rays are a hoax"
"Air travel will never amount to anything"
"Everything that can be invented has been invented"
....and so forth.

One last point: Many people read schematics and have a remarkable knowledge of what a device does, and how, from that reading. Many musicians have the corresponding skill. Not just Beethoven or Mozart.

And just as a performance's interpretation can change how we perceive that piece of music, so can _my_ implementation of a UHF amplifier suddenly become an oscillator.

So, perhaps as we pursue the wonders of antennas, we can also make allowances for our - hopefully improving - abilities to implement new designs.

We need all kinds of thinkers. The crazy idea is only one, which hasn't been made to work yet.

Our ability to model and evaluate analog electronics is only as good as our ability to know and express all its complex factors.

In the end, the performance is what matters, not the script, nor the dress rehearsal.

As always, kindest regards,
Jim

Jim: Thanks for this response.

I thought your reply was very good and should be shared with the list. It really makes the point I was striving for in a roundabout way — and, that it is good to have skills of all kinds to come up with extraordinary answers. We *need* technical abilities based on the teachings of the masters, but we also *need* those who can think outside the "can".....

We need those who can look at an antenna and explain it on the chalkboard — and then we need those who can simply look at the antenna and simply explain it.

This list has those ingredients IMHO.....
Best regards,
Jack L. Stone

….and the discussion continues at:
http://www.antennex.com/listlogin/

Great minds at work!

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