
3 Notes on Transmission Lines

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NOTE 1. Another Approach to Transmission-Line Analysis

Introduction

The subject ‘Transmission Lines’ has had a mist of mystery to the world ham community (and even to some professionals). The reason for this is that the concepts involved are not well presented, permitting some degree of speculation and, in my opinion type discussions, generating polemics.

In texts on transmission lines, a generator normally it is said ‘matched with a line’ when its output impedance is equal to Z_o , the surge impedance of the line. This is not a real match when we remember that, in all well-behaved cases¹, a generator is matched with the load when the former transfers the maximum power to the latter, that is, its output impedance is the complex conjugate² of the impedance seen at the load. In the line general case, that impedance is not Z_o and, therefore, we will consider here that a generator is matched to a line when the input impedance of the latter (resulting from the load impedance at the upper end of the line) is matched to the generator output impedance and Z_o is merely a parameter of the line itself. For us, here, “matched to the line” and “matched to Z_o ” are two completely different things and must be well understood because they will be used from now on in the article.

The “match to the line” is more coherent with the following situation: suppose we have a generator connected to a black box (there may exist a line within the box, but we don’t know). We ask someone to adjust the generator output impedance to match the black box input. He cannot adjust it to Z_o because he was not told about the details of the box. He will adjust things to get the maximum power transfer to the box and this will occur when the generator is matched to the ‘seen’ input impedance of the black box.

We will not present here any derivation or concept already commonly presented in other texts. As “return loss” and “line loss” are independent things, we will divide our text into two parts, ideal lines and lossy lines.

Ideal Lines

To simplify things, we will consider for the time being that the line is ideal (lossless) and the load at the upper end of it is pure resistive³. Let’s have a generator connected to a line (Z_o) and this to a load $R_i = Z_o$ as in **Figure 1**.

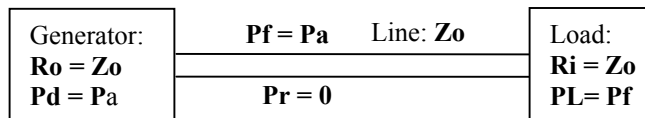


Figure 1

¹ Linear and time independent circuits, as normal lines are.

² Or simply equal, when no reactances are involved.

³ This will not lead to a loss of generality; it will only simplify the discussions.

As the input impedance of the load is $\mathbf{R}_i = \mathbf{Z}_o$, the reflected power \mathbf{P}_r is null and the impedance seen by the generator is \mathbf{Z}_o . This delivers a power \mathbf{P}_d equal to the maximum available power \mathbf{P}_a entirely to the line and the forward power \mathbf{P}_f is equal to \mathbf{P}_a . Then the power \mathbf{P}_L delivered to the line is \mathbf{P}_a too. This is the simple case of entire matching.

Now suppose we keep all conditions but the impedance of the load, that is now $\mathbf{R}_i = \mathbf{Z} \neq \mathbf{Z}_o$, as in **Figure 2**. We have a reflected power \mathbf{P}_r (not null) that goes towards the generator. Suppose we use an ideal coupler, (if necessary) to match the generator with the line (not with \mathbf{Z}_o), that is, all maximum available power is kept delivered to the line.

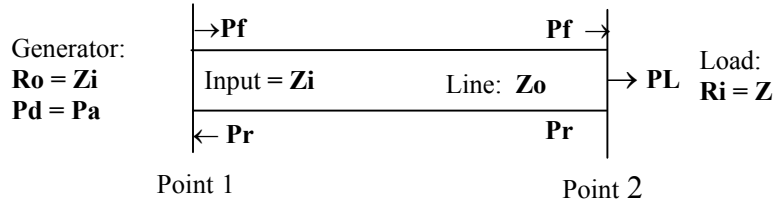


Figure 2

Remembering that, in any non-dissipative point of a circuit, the arriving power must be equal to the leaving power (by energy conservation law), we may analyze what happens on Points 1 and 2 of the figure (they are respectively the generator and the load ends of the line).

On Point 1, the arriving power is $\mathbf{P}_r + \mathbf{P}_a$ and the leaving power is \mathbf{P}_f , so $\mathbf{P}_f = \mathbf{P}_a + \mathbf{P}_r$ **(I)**. On point 2, the arriving power is \mathbf{P}_f and the leaving power is $\mathbf{P}_L + \mathbf{P}_r$, so $\mathbf{P}_f = \mathbf{P}_L + \mathbf{P}_r$ **(II)**. By comparing **(I)** and **(II)**, we get $\mathbf{P}_L + \mathbf{P}_r = \mathbf{P}_a + \mathbf{P}_r$, or $\mathbf{P}_L = \mathbf{P}_a$, independently of the reflection coefficient, as we have not mentioned it. That means with a matched generator to the line all the delivered power from the generator is dissipated in the load, no matter how big is the mismatch between the line and the load. As all the involved powers are positive numbers (square of voltages divided by positive impedances or square of currents multiplied by positive resistors), from **(I)** we can see that the forward power \mathbf{P}_f is greater than the available one, \mathbf{P}_a .

A question arises immediately: isn't it a power creation from nothing? The answer is NO, because the forward power is fed not only by the generated power, but also by the reflected one; the excess of power on Point 2 that is not delivered to the load, just \mathbf{P}_r , will contribute to \mathbf{P}_f on Point 1, as in Figure 2. We may say that it exists a power equal to \mathbf{P}_a (the maximum available power) going through the line to the load and dissipating in it, plus a circulating power \mathbf{P}_r . The forward part of this circulating power sums with \mathbf{P}_a in the line to form \mathbf{P}_f and its backward part is the power \mathbf{P}_r itself (it is a mesh analysis), as in **Figure 3**.

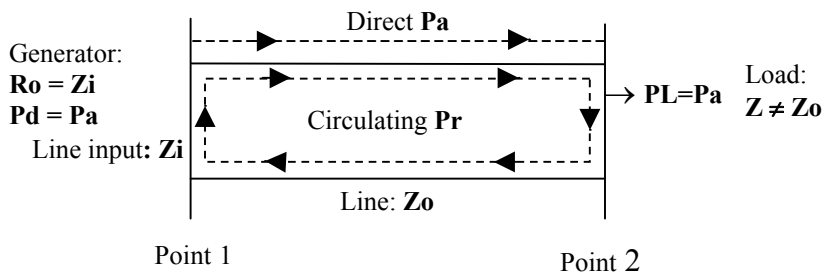


Figure 3

This shows that **it is not** the reflected power itself that is the reason of the efficiency decrease when there exists a mismatch between a lossless line and the load, as all the available power is delivered to it when we are dealing with lossless lines. This is perfectly in agreement with the fact that, if all power is delivered (nothing given back) by the generator to a system composed by a lossless element (line) and a lossy element (load), this power can be only dissipated in the load (by energy conservation), nothing said about reflections.

When we don't have a "match to the line" condition at the generator end of the line, but, instead, a "match to **Z_o**" situation, we have indeed a generator mismatch and **this** is the real reason for the loss, that, despite being called "return loss", it is an internal generator loss.

Lossless lines are only lossless impedance transformers, with the reflected power being used only to adjust and match the involved impedances. **Z_o**, although having an impedance dimension, is only a parameter of the transformer.

Lossy Lines

In the real world, however, the presence of a reflected wave increases the voltage/current peaks in certain points of the line (standing waves, with peaks and valleys) and these peaks increase the losses⁴. So it is the **line loss** that is responsible for the losses in a mismatched line system and **not** the reflection itself, as commonly thought by many people.

As a simple and well-known example, is the case of a 50-Ohm resonant antenna fed with a lossless 300-Ohm $\frac{1}{2}$ wavelength long line. The impedance seen by the 50-Ohm generator is also 50 Ohm because of the line length, but we have a mismatch of 6:1 at the antenna end. Even with a great reflection occurring, no loss will happen. We see that, despite of the difference between generator (50 Ohm) and line (300 Ohm) impedances, the reflected power doesn't mean any loss and no "return loss" occurs. It is just because the generator is really matched to the input impedance of the line and not to its surge impedance (when nothing is told about power transfer).

"Matching to **Z_o**"

If the line is lossless and the generator is "matched to the line" (not to **Z_o**), we see that, even existing a reflected power, there is no **return loss**. The latter is important, however, when there is a mismatch between the line and the load and we have the condition of the generator "matched to **Z_o**". The reflected power sees the impedance **Z_o** of the generator when it arrives to the lower end of the line and, therefore, is totally transferred to the generator that **dissipates** it. The forward power is the same of the fully matched case, but the power transferred to the load is the forward power minus the reflected one. So, we have less power on the load and this difference is dissipated in the generator. Now we have a so-called "return loss". When we have a lossless coupler at the generator output and get a "match to the line" condition, the coupler nullifies that return loss, even with high load mismatch. That's why the term "return loss" perhaps is not very convenient and it is one of the reasons for the general misunderstanding about transmission lines.

⁴ Due the quadratic behavior of the power, the loss increase on the peaks is higher than the loss decrease on the valleys, so the net loss is higher.

Conclusion: Note 1

When we have a transmitter with fixed output impedance with no coupler to a lossless line, it is advisable to match the latter with the antenna because of the return loss. With a coupler, or variable transmitter output impedance, the VSWR value is irrelevant for those lines.

In the real world, where lines have losses that increase with VSWR, we must keep the latter as low as possible in any situation, but couplers are still useful to cancel the return loss⁵.

I guess that this focusing of the subject (embracing the fact that the forward power is greater than the maximum available one in the general lossless case and the real reason for the losses in the mismatched cases) is really a novel one. I have never seen it explicitly in the literature.

⁵ We see clearly here that the expression “return loss” is not very convenient because the coupler, at the generator end of the line, is able to cancel the return loss without affecting the returning (reflected) power in the line that depends only upon the matching conditions at the its antenna end. This article doesn’t intend to change the use of the expression “return loss” in the literature as it is a well-accepted term, but only call the reader’s attention to its correct concept.

NOTE 2. Transmission Lines in Parallel

This article shows the result when we put in parallel two ideal transmission lines with any surge impedances and with any type of load, but with the same electrical length, that is, lengths taking into account the velocity factors of the lines.

A generator with voltage V is connected to two transmission lines with the same electrical lengths in parallel, that is, loaded by one common impedance Z_c , as in **Figure 1**. The surge impedances of the lines are Z_{o1} and Z_{o2} . The generator ‘sees’ a reflected impedance Z_r . On the load, the voltage is V_c .

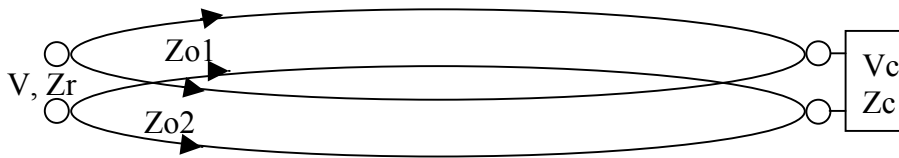


Figure 1

To analyse what happens, we separate the system into two loads Z_{c1} and Z_{c2} , one for each line, so that this configuration results the same impedance Z_r to the generator.

With the lines put in parallel, we get the circuit of **Figure 1**. In other words, we create a new circuit in such a way that it doesn't matter if the loads Z_{c1} and Z_{c2} are paralleled or not under the generator point of view. The system with separated loads is showed in **Figure 2**.

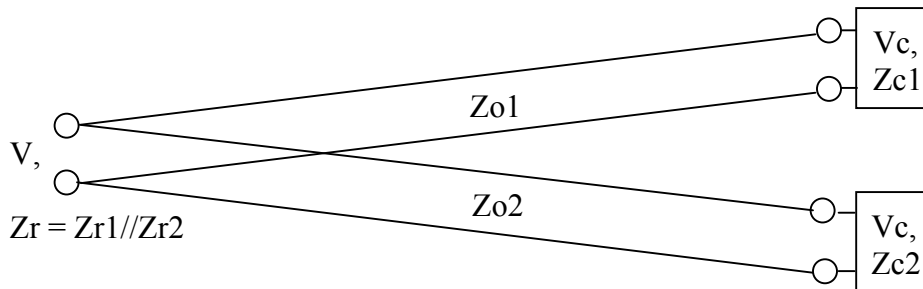


Figure 2

In this scheme, the load Z_{c1} reflects Z_{r1} to the generator through the line with surge impedance Z_{o1} and Z_{c2} reflects Z_{r2} to the generator through the line with impedance Z_{o2} . Z_{r1} in parallel with Z_{r2} is equal to the original reflected impedance Z_r .

As we want that the parallel of Z_{c1} and Z_{c2} is equal to the original load Z_c , the phases of Z_{c1} and Z_{c2} must be equal, that means, both have the same reactance/resistance ratio. Besides, as explicit in Figure 2, we need that the voltages on the loads are equal to V_c in amplitude and phase, as it is the original voltage on the load Z_c . This guarantees that the two loads Z_{c1} and Z_{c2} can be put in parallel not affecting the generator. So, we may write:

$$Z_c = Z_{c1} \cdot Z_{c2} / (Z_{c1} + Z_{c2}) \quad [1]$$

If the voltages on the loads are equal, these loads are proportional to the corresponding surge impedances. So:

$$Z_{c1} / Z_{o1} = Z_{c2} / Z_{o2} \text{ ou } Z_{c2} = Z_{o2} \cdot Z_{o1} / Z_{c1} \quad [2]$$

But $Z_c = Z_{c1} // Z_{c2}$:

$$Z_c = Z_{c1} \cdot Z_{c2} / (Z_{c1} + Z_{c2}) \quad [3]$$

Applying [2] in [3], we get:

$$Z_c = Z_{c1} \cdot Z_{o2} / (Z_{o2} + Z_{o1}) \quad [4]$$

$$\text{or } Z_{c1} = Z_c \cdot (Z_{o2} + Z_{o1}) / Z_{o2} \quad [5]$$

the same for Z_{c2} :

$$Z_{c2} = Z_c \cdot (Z_{o2} + Z_{o1}) / Z_{o1} \quad [6]$$

For ideal lines, the reflected impedance z_r , for given surge impedance z_o and load z_c , is given by:

$$z_r = z_o \cdot (z_c + z_o \cdot t) / (z_o + z_c \cdot t), \text{ onde } t = j \cdot \text{tg } \beta \cdot L$$

Here j is the imaginary unity and $\beta = 2 \cdot \pi / \lambda$, with λ being the wavelength in the line, that is, taking into account the velocity factor of the line. Applying the last expression for lines 1 and 2, we have:

$$Z_{r1} = Z_{o1} \cdot (Z_{c1} + Z_{o1} \cdot t) / (Z_{o1} + Z_{c1} \cdot t) \quad [7]$$

$$Z_{r2} = Z_{o2} \cdot (Z_{c2} + Z_{o2} \cdot t) / (Z_{o2} + Z_{c2} \cdot t) \quad [8]$$

Replacing in [7] e [8] Z_{c1} e Z_{c2} by their values of [5] and [6], we have:

$$Z_{1r} = Z_{o1} \cdot [Z_c + t \cdot Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] / [Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o}) + Z_c \cdot t] \quad [9]$$

$$Z_{2r} = Z_{o2} \cdot [Z_c + t \cdot Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] / [Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o}) + Z_c \cdot t] \quad [10]$$

Let's write:

$$P = [Z_c + t \cdot Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] / [Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o}) + Z_c \cdot t] \quad [11]$$

So:

$$Z_{1r} = Z_{o1} \cdot P \quad [12]$$

$$Z_{2r} = Z_{o2} \cdot P \quad [13]$$

As we want that Z_{1r} in parallel with Z_{2r} results in Z_r , we have:

$$Z_r = Z_{1r} \cdot Z_{2r} / (Z_{1r} + Z_{2r}) \quad [14]$$

Putting [12] e [13] em [14], we get:

$$Z_r = Z_{o1} \cdot Z_{o2} \cdot P / (Z_{o1} + Z_{o2}) \quad [15]$$

Now we want to know which is the surge impedance of the line that, alone, replaces the paralleled line pair, resulting in the same impedance Z_r with the load Z_c . By using the general expression for the reflected impedance, we have:

$$Z_r = Z_o \cdot (Z_c + Z_o \cdot t) / (Z_o + Z_c \cdot t) \quad [16]$$

Repalcing P in [15] by its definition in [11], we get:

$$Z_r = [Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] \cdot \{Z_c + [Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] \cdot t\} / \{[Z_{1o} \cdot Z_{2o} / (Z_{1o} + Z_{2o})] + Z_c \cdot t\} \quad [17]$$

The expression [15] corresponds to a unique line and [17] to the case of two paralleled lines. Comparing both, we see that:

$$Z_o = Z_{o1} \cdot Z_{o2} / (Z_{o1} + Z_{o2}) \quad [18]$$

[18] shows that the surge impedance Z_o of the line that repalaces the pair of lines in is just the pallel of the surge impedances Z_{o1} e Z_{o2} .

We can say finally: “If a generator is connected to a load through a pair of ideal transmission lines in parallel with surge impedances Z_{o1} and Z_{o2} , both with the same electrical length, the unique ideal line with the same electrical length that replaces that system has its surge impedance Z_o equal to the pallel of Z_{o1} and Z_{o2} .”

Example: We want a piece of cable that transforma the 25 Ohm input impedance of an antenna into 50 Ohm to use any length of 50 Ohm cable to the transmitter. An $\frac{1}{4}$ wavelength line would do the job and its surge impedance would be given by:

$$Z_o^2 = 25 \cdot 50 \text{ Ohm}, \text{ or, } Z_o \approx 36 \text{ Ohm}.$$

This cable doesn't exist commercially, but two pieces of a 73 Ohm cable in parallel would result in a simulated cable with 36 Ohm and would solve the problem. In this example we have perfect impedance match and the load is pure resistive, that is, real. In the general studied case it is not necessary, the load and the electrical length may have any values..

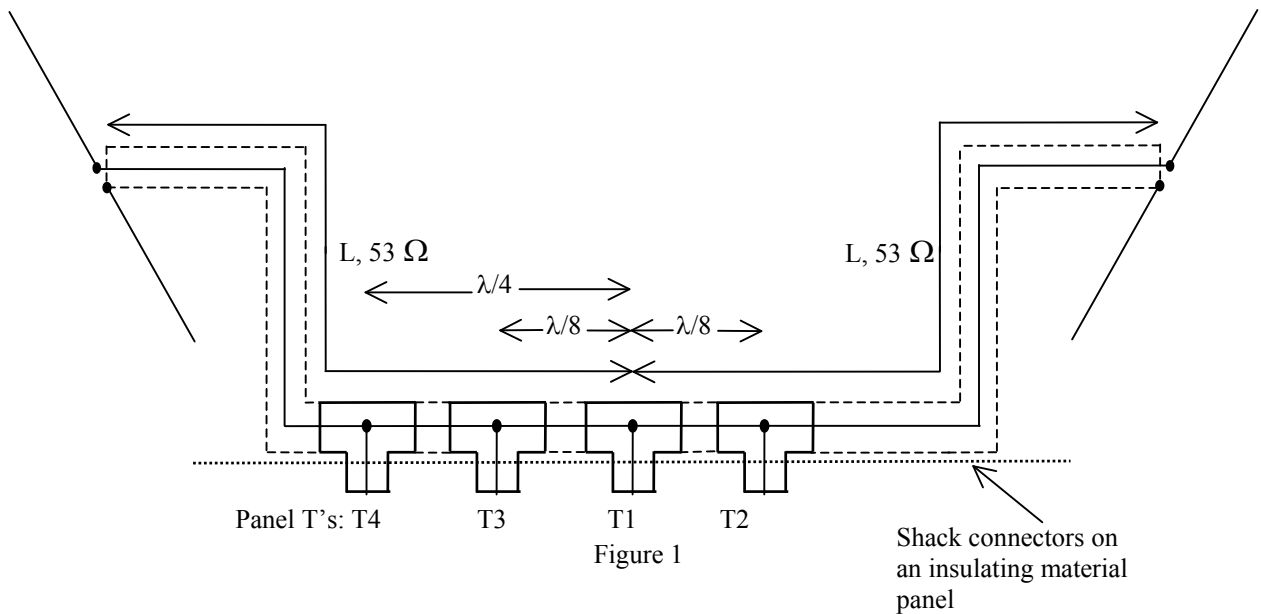
End Note 2

NOTE 3. A Method to Connect Crossed-Yagis Getting 4 Different Polarizations

For those who have their antennae rather near their radio shack, this seems to be a good solution for getting the 4 possible polarizations using two VHF/UHF crossed-yagis (on the same boom or not). Normally, to do that connection, we have to use pieces of $1/4$ wavelength (odd multiples) of 73-Ohm lines and relays or some other remote control system if we want to change from the original wave polarization.

Here I describe a system that only uses, connected to the antennae, 53-Ohm coaxial cables and all the polarization switching is done in the shack, with no remote control or relays. The price one pays is that we have to use two coaxial pieces from the shack to the antennae. The drawing, to be easily read, represents physically separated yagis but they may be assembled on only one boom.

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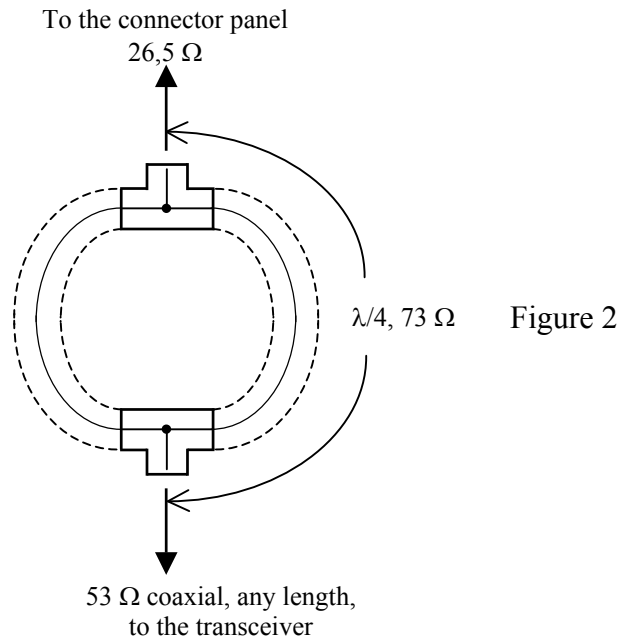


Figure 2

A piece of 53 Ohm coaxial with length $2L$ (any) is connected between the two antennæ in such a manner that it comes from one antenna, passes through the shack and returns to the other antenna as in **figure 1**.

The antennæ must be mounted so that they are mutually perpendicular and 45° with the horizontal each (if they are mounted on the same boom making a real cross-yagi, their corresponding elements must form a 'X' rather than a '+' in the space).

In the shack we use a panel made of an insulating material¹ with 4 (four) coaxial T's put on it (T1, T2, T3 and T4). T1 is exactly at the center of the cable, that is, it is equidistant (distance L) from both antennæ. As the upper part of both dipoles are connected to the same cable conductor (center conductor in our case of **figure 1**), the connection of a transmitter to connector T1 will produce currents in phase on both antennæ with vertical polarization.

As T4 is $\lambda/4$ away from the center, the currents at the two antennæ will now be 180° out of phase and the polarization will be horizontal when power is applied to this T. As T2 and T3 are $\lambda/8$ away from the center, the currents at the two antennæ will be 90° out of phase and, as the amplitudes are the same (neglecting coaxial losses), we have circular polarization in either direction. The impedance is 26.5Ω for any of the T's because the cable is perfectly matched at both ends.

The problem that arises is how to match a 26.5Ω point to a 53Ω coaxial cable to the transceiver. We see that this will be easily done with a $36.5 \Omega \lambda/4$ line because $36.5^2 \cong 26.5 \times 53$.

¹ A conducting panel would short-circuit the cables shields.

It can be easily shown² that two line pieces with the same electrical length put in parallel are equivalent to a single piece of line with the same electrical length, but with the characteristic impedance Z_0 following the parallel resistance law, that is, $1/Z_0 = 1/Z_1 + 1/Z_2$, where Z_1 and Z_2 are the impedances of the two line pieces. So, if we put two pieces of $\lambda/4$ 73 Ω in parallel (to use T's, as in figure 2, is a good idea), we get just an equivalent piece of $\lambda/4$ 36.5 Ω line.

Thus, by changing the T where the impedance transformer of figure 2 is connected on the panel, we can choose among the 4 possible polarizations **at the shack**, with no relays or remote control.

End Note 3

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BRIEF BIOGRAPHY OF AUTHOR



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Luiz was Born in the city of Rio de Janeiro, Brazil in December, 1941. He holds degrees as Bachelor and Master in Physics (nuclear).

Luiz is a ham operator since 1958, with both callsigns PY1LL and PY4LC, having many works in brazilian and international publications and many patents among pending and deferred.

Now retired and writer of several articles on radio, electronics and mathematics and living on a ranch where it is possible to experiment with antennae, especially on low frequency HF and is

his preferred challenge.

He has two daughters, one son, four grandchildren and a dog.

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² See the article “Transmission Lines in Parallel” above.